Reduced Order Models for Bifurcation Problems in Computational Fluid Mechanics with Autonomous Localization through Machine Learning

Martin Hess[a] and Annalisa Quaini[b] and and Gianluigi Rozza[a]

[a] *SISSA, International School for Advanced Studies, Mathematics Area, Trieste, Italy*
[b] *Department of Mathematics, University of Houston, Houston, USA*

We consider channel and cavity models in computational fluid dynamics (CFD), undergoing bifurcations of steady (pitchfork bifurcations) and unsteady (Hopf bifurcations) type. Physical parameters, such as Reynolds number or Grashof number as well as parametric variation in geometry are considered. This is work based on results published in [1, 3, 4] and numerical simulations are based on the spectral/hp element method [2].

Since field solutions corresponding to varying parameters show commonalities across parts of the parameter domain, while differing strongly over larger areas of the parameter doamin, we use classical clustering techniques to automatically identify regions of similar field solutions. Each such region then defines a distinct localized reduced order model (ROM). When solving *online* for a new parameter of interest, only the most relevant localized ROM needs to be evaluated. This procedure allows for small sized, accurate ROMs, in contrast to global ROM approaches.

During this investigation, it turned out that the cluster selection criterion for a new parameter of interest is crucial to the overall accuracy. This is especially true when looking at higher dimensional parameter domains (≥ 2). We identified a neural network (NN) training procedure, which allows to gain an order of magnitude in accuracy over straightforward cluster selection approaches, such as *distance to parameter centroid* or *distance to next snapshot.* This gain is in the sensitive error regime of practical interest, e.g., it improves the relative L^2 error in the fluid velocity from 5% to 0.5% in the channel model undergoing a pitchfork bifurcation with changing Reynolds number and geometry.

We also investigate how the NN performs against classical regression techniques, such as Kriging or rational interpolation. The computational times of all cluster selection techniques are well below typical offline-online compute times in model reduction.

References

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